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On the (Frequency) Modulation of Coupled Oscillator Arrays in Phased Array Beam Control

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It has been shown that arrays of voltage controlled oscillators coupled to nearest neighbors can be used to produce useful aperture phase distributions for phased array antennas. However, placing information on the transmitted signal requires that the oscillations be modulated. Frequency (phase) modulation is the most natural for such systems. The modulating signal is applied to the tuning ports of the oscillators producing a frequency shift on which is coded the information to be transmitted. The theory of such arrays predicts the transient behavior of the array under modulation and the present paper reports on efforts to verify these theoretical predictions experimentally.
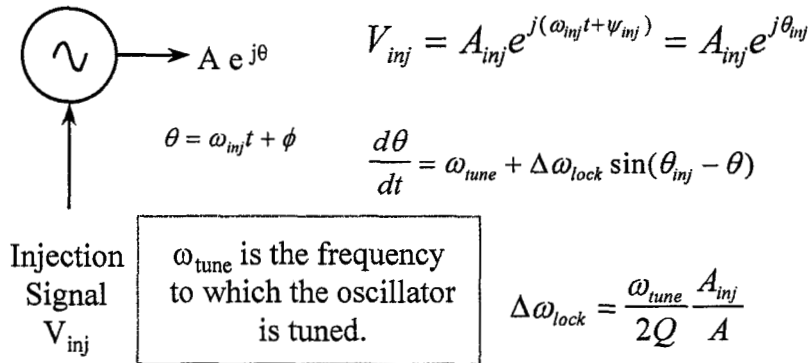
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Agenda

- **Background**
  - Coupled Oscillator Arrays.
  - Beam Steering via Coupled Oscillator Phase Control.
- **Modulation**
  - Theoretical results.
  - Experimental results.
- **Concluding Remarks**

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We begin with a short description of the theory of coupled oscillator arrays and the concept of using such arrays to achieve beam steering in phased array antennas. Next we treat the case of square wave frequency modulation of the array theoretically and show that modulation of any but all of the oscillators will be ineffective in transmitting the information. Experimental observations corroborating these theoretical predictions are then presented and conclusions concerning the manner in which information can be effectively transmitted are drawn.

Injection Locking

$$V_{inj} = A_{inj}e^{j(\omega_{inj}t+\psi_{inj})} = A_{inj}e^{j\theta_{inj}}$$

$A\,e^{j\theta}$

$\theta = \omega_{inj}t + \phi$

$$\frac{d\theta}{dt} = \omega_{tune} + \Delta\omega_{lock}\sin(\theta_{inj}-\theta)$$

Injection
Signal
V_{inj}

ω_{tune} is the frequency to which the oscillator is tuned.

$$\Delta\omega_{lock} = \frac{\omega_{tune}}{2Q}\frac{A_{inj}}{A}$$

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Consider a single injection locked oscillator. We represent the signals as complex functions as indicated. The injection signal is V_{inj} in which A_{inj} is the amplitude and ω_{inj} is the radian frequency. Similarly the output signal amplitude is A and the phase is θ which is the sum of the phase due to oscillation, $\omega_{inj}t$, and the relative phase, ϕ. $\Delta\omega_{lock}$ is the radian locking range which, as shown, depends on the Q of the oscillator and the strength of the injection signal relative to the output. In steady state, of course, the oscillator will oscillate at the injection frequency. The transient (time varying) behavior is governed by the indicated differential equation. Using this equation we can formulate the theory of a set of coupled oscillators.

JPL Coupled Oscillators
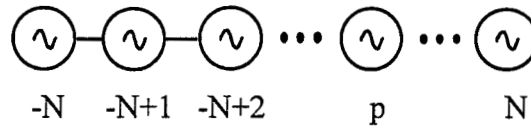
$$\frac{d\theta_i}{dt} = \omega_{tune} - \frac{\omega_i}{2Q} \sum_{\substack{j=i-1 \\ j\neq i}}^{j=i+1} \varepsilon_{ij} \frac{A_j}{A_i} \sin(\Phi_{ij} + \theta_i - \theta_j)$$

In the continuum model:

$$\nabla^2 \theta - \frac{\partial \theta}{\partial \tau} = -\frac{\omega_{tune}}{\Delta \omega_{lock}}$$

Here we adapt the preceding differential equation to describe the behavior of a linear array of coupled oscillators with nearest neighbor coupling. Using a continuum model of this description leads to the partial differential equation shown at the bottom of the vugraph. Tau is time multiplied by the locking bandwidth of the oscillators.

JPL Coupled Oscillators
(Continued)

Define the phase of the ith oscillator, ϕ_i, by:

$$\theta_i = \omega_{ref} t + \phi_i$$

Then,

$$\frac{\partial^2 \phi}{\partial x^2} - \frac{\partial \phi}{\partial \tau} = -\frac{\omega_{tune} - \omega_{ref}}{\Delta\omega_{lock}} = -Cu(\tau)\delta(x-b)$$

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We define the phase of the ith oscillator with respect to a reference frequency to be selected to be the initial ensemble frequency of the array which has been shown to be the initial average of the oscillator tuning frequencies. In the case where the oscillator at x=b is detuned by C (measured in locking ranges), the partial differential equation we wish to solve take the form shown.

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The Finite Array

Boundary conditions can be derived from,

| -a-1 | -a | -a+1 | -a+2 | b | a | a+1 |
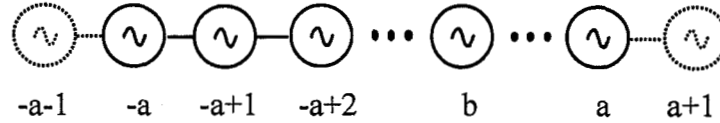
Tune added oscillators so that, $\phi(-a-1) = \phi(-a)$

Then, $\left.\dfrac{\partial \phi}{\partial x}\right|_{-a-\frac{1}{2}} = 0$ $\phi(a+1) = \phi(a)$

$\left.\dfrac{\partial \phi}{\partial x}\right|_{a+\frac{1}{2}} = 0$ **That is, the classical Neumann conditions.**

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To find the solution corresponding to an array of finite length, 2a, one must effectively add homogeneous solutions of the equation to the particular integral in sufficient amounts to satisfy the boundary conditions at the ends of the array. These boundary conditions can be ascertained using the artifice indicated here. That is, two fictitious oscillators are added to the array, one at each end. These oscillators are assumed to be dynamically tuned in such a manner as to maintain their phase equal to the phase of the corresponding actual end oscillator. This condition assures that no injection effect is transmitted between these pairs of oscillators. This shows that the correct boundary condition is one of classical Neumann type applied one half unit cell outside each end of the array.

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Beamsteering Dynamics

Equal and opposite detuning of the end oscillators; i.e.,

$$\Delta\omega_L = -\Delta\omega_R = \Delta\omega_T$$

yields,

$$\phi(x,\tau) = \frac{\Delta\omega_T}{\Delta\omega_{lock}} \sum_{m=0}^{\infty} \frac{2\sin\left(b\sqrt{\sigma_m}\right)\sin\left(x\sqrt{\sigma_m}\right)}{(2a+1)\sigma_m}\left(1-e^{-\sigma_m\tau}\right)$$

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According to Liao, et.al. [IEEE Trans. MTT-41,pp. 1810-18115, Oct. 1993], beamsteering is accomplished by equal and opposite detuning of the end oscillators of the array. The solution for the phase distribution can be obtained from the solution for detuning one arbitrary oscillator (x=b) by superposition (subtraction) of two solutions, one for b=a and one for b=-a. The time domain result is as shown.

Beamsteering Phase
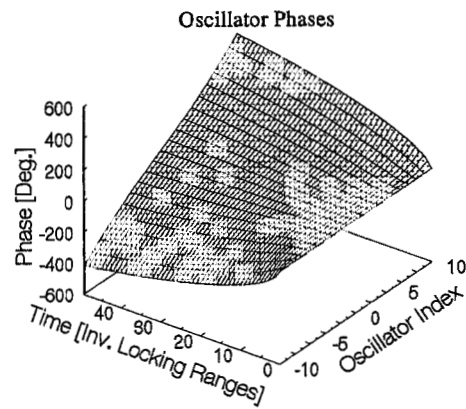
This is a graphical representation of the beamsteering phase solution just obtained.

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Far Zone Radiation Pattern

Scanning Beam
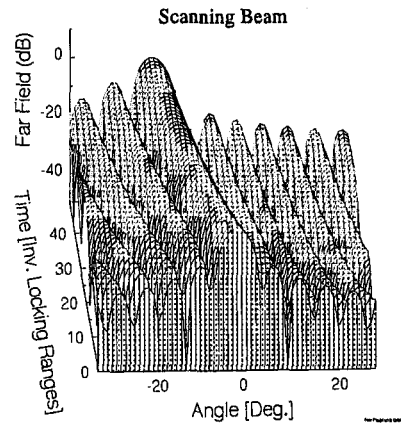
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This plot shows the dynamics of the far zone radiation pattern during beamsteering. It was obtained by computing the radiation pattern for each time value by integration over the aperture using the phase solution represented on the previous vugraph. Note that the beam integrity and sidelobe structure is maintained throughout the transient period.

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Steady State Steered Beam

$$\omega(x) = \omega_o + \Delta\omega_L\,\delta(x+a) + \Delta\omega_R\,\delta(x-a)$$

$$\phi(x) = \left(\frac{\Delta\omega_L + \Delta\omega_R}{2\Delta\omega_{lock}}\right)\left(\frac{x^2 + b^2}{2a+1} + \frac{2a+1}{6}\right)$$
$$- \left(\frac{\Delta\omega_L - \Delta\omega_R}{2\Delta\omega_{lock}}\right)x$$

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In steady state the phase distribution across the array is parabolic. The even part is controlled by the sum of the left and right end tuning while the odd (linear) part is controlled by their difference. Thus, for equal and opposite detuning the distribution is linear which is, of course, just that needed to produce a well formed off-axis (steered) beam.

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Frequency Modulation

- Appear to be three options for frequency modulation.
  - Modulate one oscillator.
  - Modulate more than one oscillator.
  - Modulate all of the oscillators.
- The first two options can be immediately discarded for the following reason.

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It will be shown in the following that to effectively place information on the radiated beam it is necessary to modulated all of the oscillators in the array simultaneously.

JPL   The Finite Array Solution

Oscillator Phases
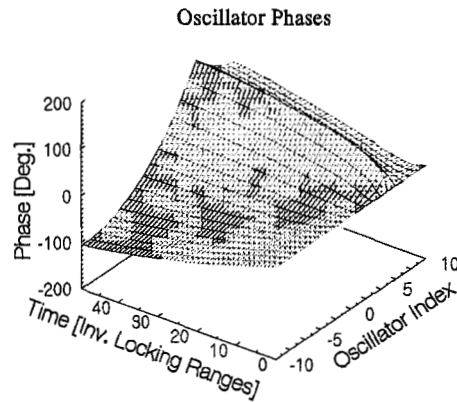
This is a graphical representation of the solution for the finite length array with oscillator "5" step detuned at t=0. Here again it is merely a plot of the analytical solution obtained via the Laplace transformation. Note that the steady state distribution is parabolic and produces a badly spoiled beam. Thus, no matter how slowly one modulates the oscillator, the beam will never reform after each switch in modulation voltage. If, on the other hand, all of the oscillators are modulated, the phase distribution will across the aperture will remain unchanged and the beam will be preserved throughout the modulation.

The Proposed Scheme
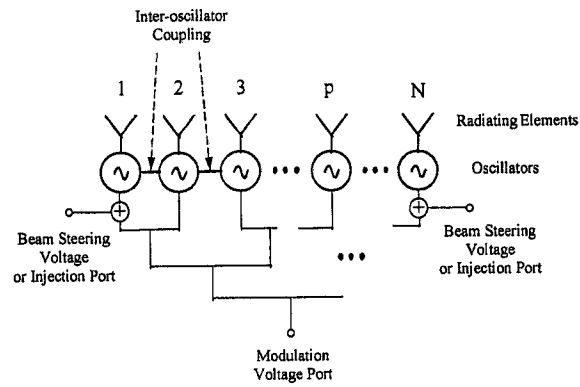
We propose to implement modulation of all the oscillators using the network shown. While resembling a corporate feed network, the line lengths here are not critical, as they would be at rf, because the network operates at baseband.

Consider a Square Wave

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The source term becomes,

$$h_{mod} = \frac{\pi}{2}u(\tau) + \sum_{n=1}^{\infty} \pi(-1)^n u(\tau - n\frac{T}{2})$$

The Laplace transform is,

$$H(s) = \frac{1}{s}\tanh\left(\frac{s}{4}\right)$$

and we wish to solve,

$$\frac{\partial^2 F}{\partial x^2} - sF = -\frac{1}{s}\tanh\left(\frac{s}{4}\right)$$

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The behavior of the proposed array under square wave modulation can be ascertained theoretically using the diffusion equation presented earlier. The source term is h_{mod}. Solution is effected via the Laplace transform. This source waveform has a known Laplace transform shown here and the resulting transformed equation is given at the bottom of the vugraph.

Square Wave Continued

A particular integral is,

$$F_p(x,s) = \frac{1}{s^2}\tanh\left(\frac{s}{4}\right)$$

Adding two complementary functions gives,

$$F(x,s) = \frac{1}{s^2}\tanh\left(\frac{s}{4}\right) + Ae^{\sqrt{s}x} + Be^{-\sqrt{s}x}$$

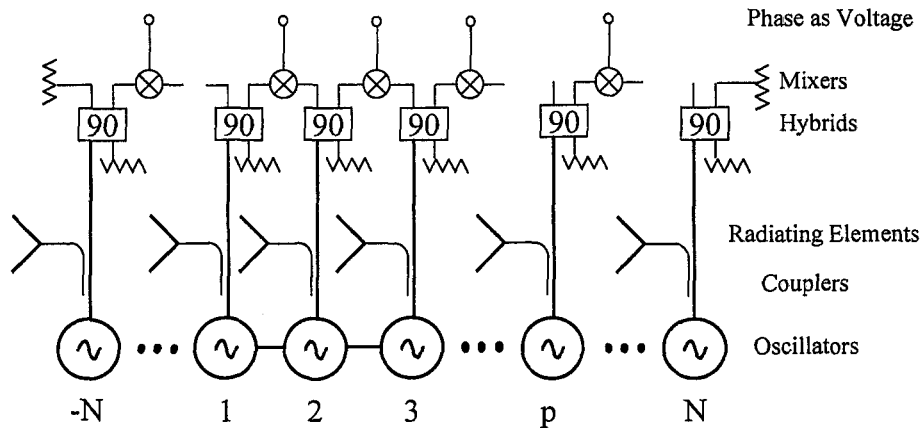Boundary conditions require that A and B be zero so the inverse transform becomes,

$$\phi(\tau) = \tau\frac{\pi}{2}u(\tau) + \sum_{n=1}^{\infty}\pi(-1)^n(\tau - n\frac{T}{2})u(\tau - n\frac{T}{2})$$

That is, the array integrates the modulation signal.

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Proceeding with the solution, we add to a particular integral the appropriate amounts of homogeneous solutions to satisfy the boundary conditions (Neumann conditions) at the array ends. The result shown here indicates that the array transmits a frequency modulated signal. For phase modulation the array basically integrates the modulation signal. Thus, we propose that in this case the information signal be first differentiated and then applied to the array modulation network resulting in transmission of an appropriately phase modulated signal.

The experimental setup for verifying the theoretically predicted array behavior makes use of mixers as phase detectors as shown here. The 90 degree hybrids are used to make the mixer outputs zero when the corresponding two oscillators are in phase. (Without the hybrids, the output would be zero for a 90 degree phase difference.) Ten dB couplers are used to derive the signals to be sent to the radiating elements. While one might expect that the mixer signals would be derived in this manner instead, the present arrangement provides adequate signal for driving the mixers while retaining the ability to measure radiation patterns since the receiver is more sensitive than the mixers.
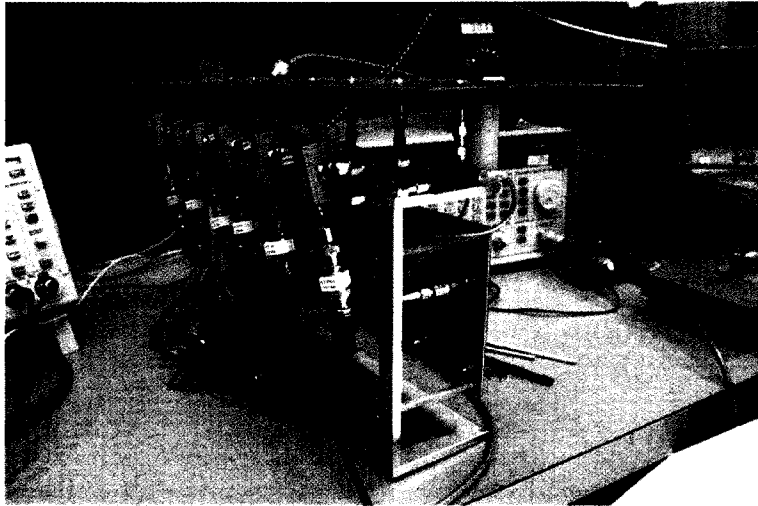
JPL Laboratory Setup
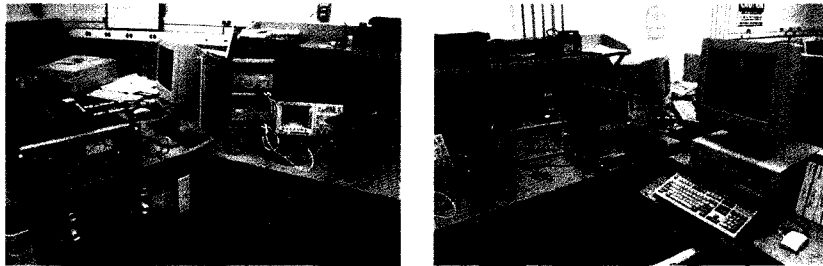
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This is a photograph of the laboratory equipment used to diagnose the array behavior.
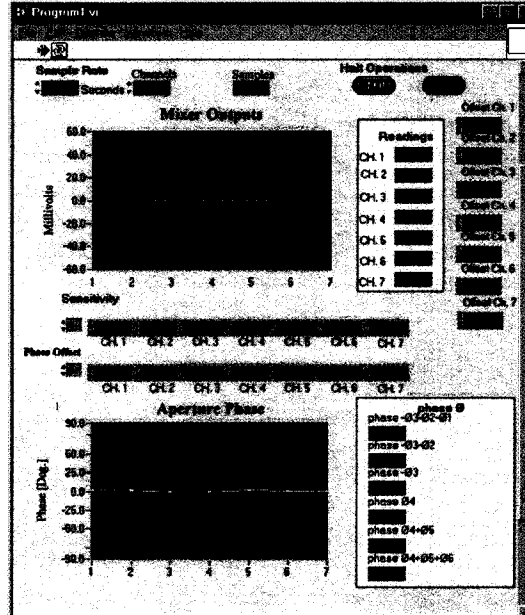
JPL   Diagnostic Circuitry

This is a photograph of the array with the added diagnostic circuitry.
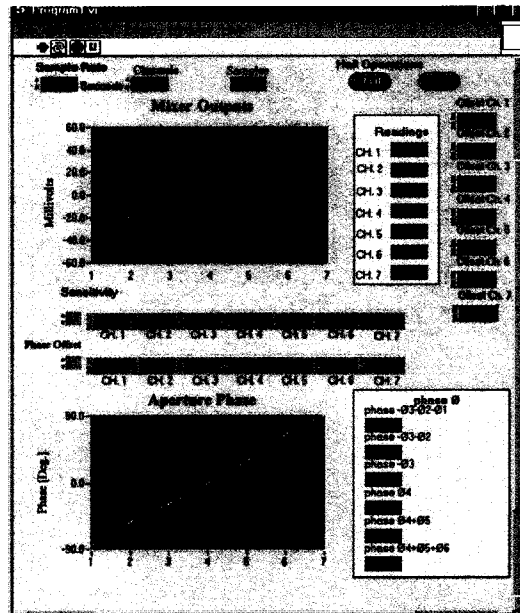
Virtual Instrument Display (In-phase Case)

The mixer outputs are read by a "Virtual Instrument" implemented in LabView. The display is shown above for tuning which yields a uniform aperture phase distribution.
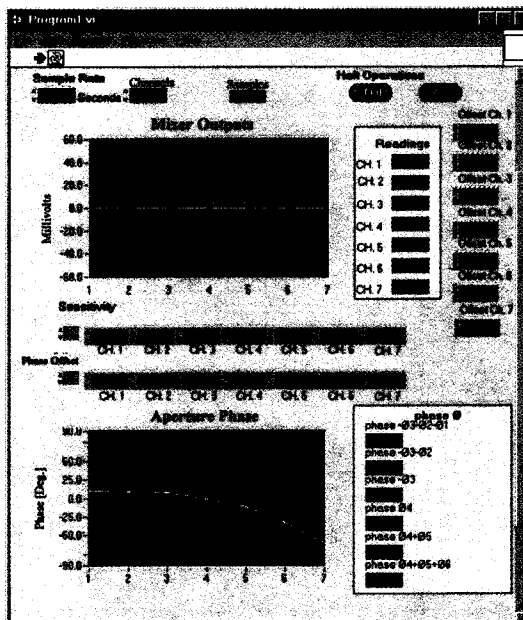
Virtual Instrument Display (Linear Phase Case)

If the end oscillators are detuned oppositely, a linear phase distribution results as shown here.
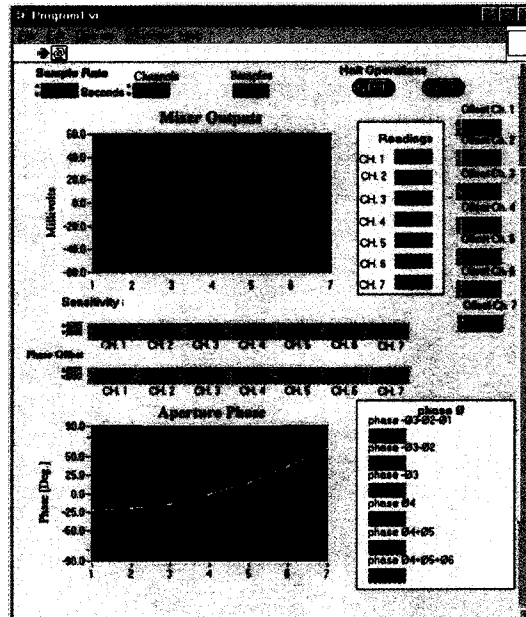
Virtual Instrument Display (Oscillator Seven Detuned)

If only oscillator seven is detuned, a parabolic distribution results.
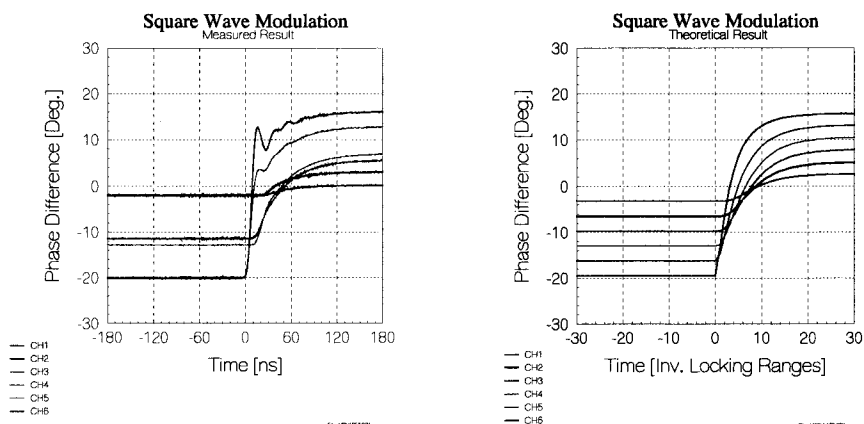
Virtual Instrument Display (Oscillator Seven Detuned)

Detuning oscillator seven in the opposite direction also produces a parabolic distribution. Modulation of oscillator seven with a square wave switches between this and the preceding distribution and permits observation of the transient behavior of the array.

Modulation of One End Oscillator

Square Wave Modulation — Measured Result

Square Wave Modulation — Theoretical Result

Implied Locking Range: 26.5 MHz

Using the system illustrated in the preceding vugraphs, the above results on the left were obtained by modulating oscillator 7 with a .2 volt peak to peak square wave injected into the oscillator tank circuit between the varactor and the resonating inductor through a very large (0.1 microFarad) capacitor. The mixers were calibrated to permit conversion of the measured output voltage to degrees of phase difference between adjacent oscillators. The corresponding theoretical prediction is shown on the right. Comparison of the plots permits confirmation of the locking range of the oscillators.
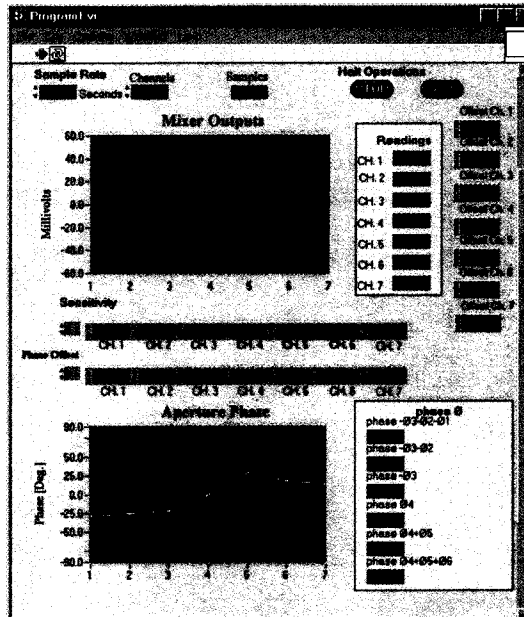
Virtual Instrument Display (Oscillator Five Detuned)
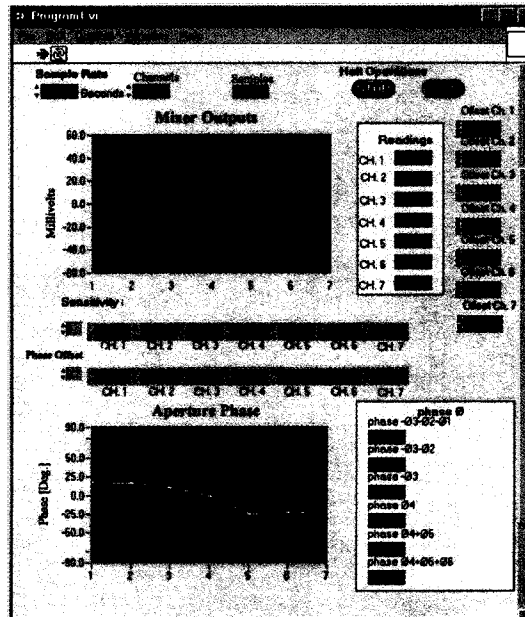
If only oscillator five is detuned, a dual parabolic distribution results.
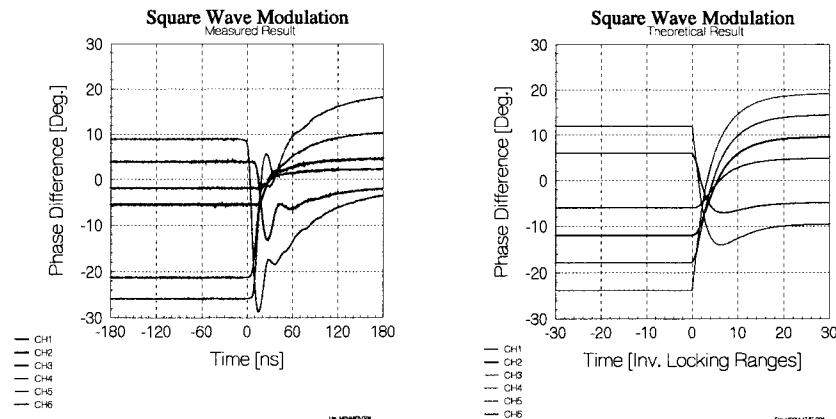
Virtual Instrument Display (Oscillator Five Detuned)

Detuning oscillator five in the opposite direction also produces a dual parabolic distribution. Modulation of oscillator five with a square wave switches between this and the preceding distribution and permits observation of the transient behavior of the array.

Modulation of One Internal Oscillator

Square Wave Modulation
Measured Result

Square Wave Modulation
Theoretical Result

Implied Locking Range: 26.5 MHz

Using the system illustrated in the preceding vugraphs, the above results on the left were obtained by modulating oscillator 5 with a .2 volt peak to peak square wave injected into the oscillator tank circuit between the varactor and the resonating inductor through a very large (0.1 microFarad) capacitor. The corresponding theoretical prediction is shown on the right. Comparison of the plots again confirms the locking range of the oscillators.

JPL Modulation of All Oscillators

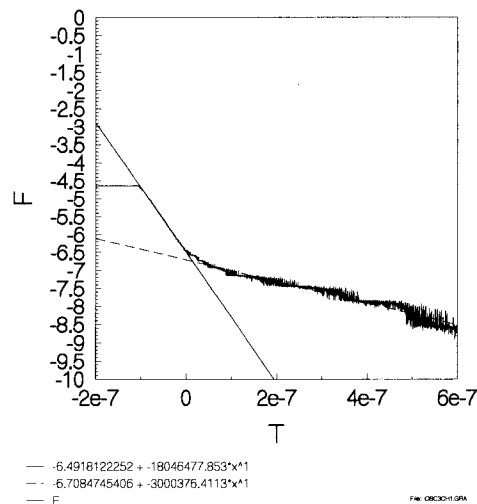Square Wave Modulation of All
Measured Result
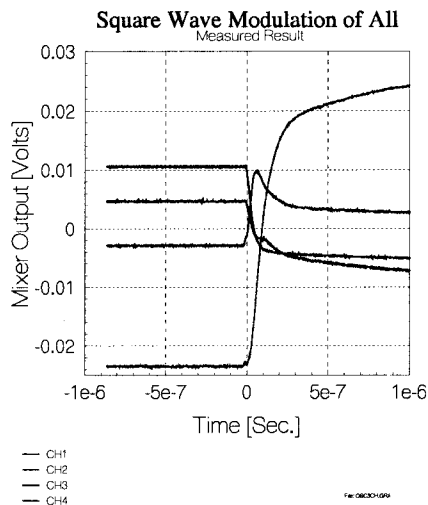
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If all of the oscillators are modulated simultaneously one should theoretically observe zero phase differences. Thus, in order to see the transients, we detuned the oscillators to produce observable phase differences and then applied the modulation to all of them. This permits the determination of the time constants from which the locking range can again be computed. Note that the short time constants are the ones relevant to the locking range computation. The long time constants are related to the large bypassing components used in the tuning power supply circuit.

Channel One
Time Constants

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Implied Locking Range:
28.725 MHz

— -6.4918122252 + -18046477.853*x^1
— - -6.7084745406 + -3000376.4113*x^1
— F

File: CBC3CH1.GRA

Selecting the channel 1 signal from the preceding graph, we fit the natural logarithm of the function to a set of straight lines whose slopes give the time constants involved. The long time constant is related to the large bypassing components used on the tuning supply. The shorter time constant implies a locking range of 28.7 MHz which is quite constant (within the experimental error) with the 26.5 MHz obtained previously and with the result of direct measurement of the locking range.

JPL   Concluding Remarks

- Modulation of one oscillator is ineffective.
  - Steady state phase distribution is parabolic.
  - Average locking range can be inferred from transients.
  - Theoretical predictions experimentally verified.
- All oscillators must be modulated.
  - Steady state phase distribution is linear.
  - Transient response is that of one oscillator.

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From the presented results, we can conclude that the theoretical predictions are born out in the measurements and that both imply that effective modulation can only be achieved by simultaneously modulating all of the oscillators in the array.